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Interactive fuzzy decision making method for solving bilevel programming problem ☆

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ABSTRACT

In this paper, an interactive fuzzy decision making method is proposed for solving bilevel programming problem. Introducing a new balance function, we consider the overall satisfactory balance between the leader and the follower. Then, a satisfactory solution can be obtained by the proposed method. Finally, numerical examples are reported to illustrate the feasibility of the proposed method.

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1. Introduction

Bilevel programming (BP) has many applications in such fields as transportation, economics, ecology, engineering and so on, and hence, it has been received increasing attention in the literature. The recent surveys and bibliographic reviews can refer to [1–11], for example.

BP involves two optimization problems where the constraint region of the upper level problem is implicitly determined by the lower level problem. The upper level decision maker (also called the leader) makes a decision first and thereafter the lower level decision maker (also called the follower) chooses his/her strategy according to the leader's action. Therefore, on the one hand, the leader's decision is able to influence the behavior of the follower without completely controlling the follower's strategy. On the other hand, the leader may be simultaneously affected by the follower's action. As a consequence, decision deadlock arises frequently and the problem of distribution of proper decision power is encountered in most of the practical decision situations.

To overcome this shortcoming, fuzzy programming methods in which both the leader and the follower have fuzzy goals for their objective functions when they take fuzziness of human judgments into consideration are presented. Lai [12] firstly introduced the concept of tolerance membership function. Thereafter, Shih et al. [13,14] extended Lai's concept by using different operator. Moreover, fuzzy programming methods were further extended by many authors to solve multilevel linear programming problems [15], decentralized two level linear programming problems [16], bilevel quadratic fractional programming problem [17], two-level nonconvex programming problems with fuzzy parameters [18], and so on. Along the line, fuzzy goal programming technique [19] was presented which overcame the shortcoming of fuzzy programming method for proper distribution of decision powers to the decision makers to arrive at a satisfactory decision for overall benefit of the

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organization. It has been studied by many authors and was extended to solve bilevel programming problems [20], multilevel programming problems [21], bilevel quadratic programming problems [22], decentralized bilevel multi-objective programming problems [23], and so on. Recently, Baky [24] also used two fuzzy goal programming procedures for solving multi-level multi-objective linear programming problems. Arora and Gupta [25] combined an interactive fuzzy goal programming approach with the concept of dynamic programming to solve bilevel programming problems. Wan et al. [26] proposed an interactive fuzzy decision making method for bilevel programming problem with a common decision variable. Wang et al. [27] extended Wan's concept to deal with bilevel multi-followers programming problem with partial shared variables among followers.

As we know, the distance function, which was proposed by Yu [28], has been widely used to solve multi-objective programming problem to achieve a compromise solution. Based on the concept of distance function, Moitra and Pal [20] used fuzzy goal programming technique and constructed a satisfactory balance via minimizing the regrets of the leader and follower as much as possible for BP. Recently, Baky and Abo-Sinna [29] proposed a fuzzy TOPSIS algorithm, which simultaneously minimized a distance function from an idea point and maximized another distance function from a nadir point, to solve bilevel multi-objective decision-making problems.

In this paper, we present a new interactive fuzzy decision making method for solving BP which is different from fuzzy programming methods mentioned above. Furthermore, we will consider the overall satisfactory balance between the leader and the follower by introducing a new balance function. Finally, numerical examples are provided to illustrate the feasibility of the proposed method. The remaining part of this paper is organized as follows. In Section 2, we describe the problem formulation. We present a new interactive fuzzy decision making method in Section 3 and give some numerical examples in Section 4, while the conclusion are given in Section 5.

2. Problem formulation

The optimistic bilevel programming problem is stated as follows:

$$\begin{aligned}
&\max_{x,y} F_1(x,y)\\
&\text{where } y \text{ solves},\\
&\max_{y} F_2(x,y)\\
&\text{s.t.}\quad G(x,y) \leqslant 0,
\end{aligned} \tag{1}$$

where $x \in R^{n_1},\ y \in R^{n_2},\ G : R^{n_1} \times R^{n_2} \to R^m$.

The decision variables of problem (1) are divided into two classes, namely the upper level decision variable x and the lower level decision variable y. Similarly, the functions $F_1 : R^{n_1} \times R^{n_2} \to R$ and $F_2 : R^{n_1} \times R^{n_2} \to R$ are the upper level and lower level objective functions, respectively.

In the following sections, we will consider problem (1) with cooperative decision makers to find a satisfactory solution.

3. New interactive fuzzy decision making method

3.1. Construction of membership functions

In the decision making context, all the decision makers are interested in maximizing their own objective functions over the same feasible region defined as $S = \{(x,y) | G(x,y) \leqslant 0\}$. The optimal solutions of them calculated in isolation can be taken as the best solution and the associated objective value can be considered as the aspiration levels of the corresponding fuzzy goals.

Denote $F_j(x,y)$ by F_j. To facilitate computation for obtaining solutions, we use the following linear membership function $\mu(F_j)(j = 1,2)$ to describe the fuzzy goals of the leader and the follower, respectively.

$$\mu(F_j) = \begin{cases} 0 & \text{if}\quad F_j < F_j^U, \\ \frac{F_j - F_j^U}{F_j^L - F_j^U} & \text{if}\quad F_j^U \leqslant F_j \leqslant F_j^L, \\ 1 & \text{if}\quad F_j > F_j^L, \end{cases} \tag{2}$$

where F_j^L (ideal value) and F_j^U (tolerance limit of the fuzzy goal) denote the values of the objective function $F_j(x,y)$ such that the degrees of the membership function are 1 and 0, respectively.

Without loss of generality, we can assume that F_j^L and F_j^U $(j = 1,2)$ are the optimal values of the following problems, respectively. For example,

$$F_j^L = \max_{(x,y)\in S} F_j(x,y) \tag{3}$$

and

$$F_j^U = \min_{(x,y)\in S} F_j(x,y). \tag{4}$$

3.2. Interactive fuzzy decision making method

In any decision making situation, achievement of highest membership function value to the extent of a fuzzy goal is always desired by a decision maker. In practice, however, it is difficult to achieve all the membership function values to the highest degree. Hence, the concept of the overall satisfactory degree between the leader and follower is advent. It is determined as follows [12]:

$$\delta = \frac{\mu(F_2)}{\mu(F_1)}. \tag{5}$$

Note that, many authors used the above formula (5) to adjust the satisfactory degree between the leader and the follower at the decision making process (for example, see [25,12,17,16,7] and the references therein). However, the situation (i.e. each decision maker's satisfactory degree may not be optimistic while the value δ of the overall satisfactory degree is large) may arise in some practical computation. Then, we consider the following balance function to measure the overall satisfactory degree which can be termed as the radio of two functions:

$$d(x,y) = \frac{\sqrt{\sum_{j=1}^{2}\left(F_j(x,y) - F_j^U\right)^2}}{\sqrt{\sum_{j=1}^{2}\left(F_j(x,y) - F_j^L\right)^2 + \sum_{j=1}^{2}\left(F_j^L - F_j^U\right)^2}}.$$

Obviously, $0 \leqslant d(x,y) \leqslant 1$ for all $(x,y) \in S$. If each decision maker achieves the ideal value, $d(x,y)$ is 1. Furthermore, $d(x,y)$ increases as the objective function values of the leader and the follower are improved. So, we can use the value of $d(x,y)$ to balance the overall satisfactory degree between the leader and follower at the decision making process.

Now, the formulation of an interactive fuzzy decision making method can be written as:

$$\begin{aligned} &\max_{x,y}\ d(x,y) \\ &\text{s.t.}\quad \mu(F_1) \geqslant \mu_1^*, \\ &\mu(F_2) \geqslant \mu_2^*, \\ &(x,y) \in S, \end{aligned} \tag{6}$$

where μ_1^* and μ_2^* are the minimal acceptable satisfactory levels specified by the leader and the follower, respectively.

Theorem 3.1. *If (x^*, y^*) is an optimal solution to problem (6), then it is also an efficient solution to problem (1).*

Proof. If (x^*, y^*) is not an efficient solution, then there exists $(\bar{x}, \bar{y}) \in S$ such that $F_j(\bar{x},\bar{y}) > F_j(x^*,y^*)$ for some j and $F_i(\bar{x},\bar{y}) \geqslant F_i(x^*,y^*),\ i = 1,2,\ i \neq j$. Therefore, we have

$$d(\bar{x},\bar{y}) > d(x^*,y^*).$$

This contradicts the fact that (x^*, y^*) is an optimal solution of (6). □

Note that, for details about efficient solution to problem (1), the reader can refer to [30,31], and so on. When the leader concludes the solution of problem (6) as a satisfactory solution, the iterative process terminates. Now, we consider the following procedure for updating the minimal acceptable satisfactory level μ_1^* (see page 92 of [7]): If the leader is not satisfied with the obtained solution and judges that it is desirable to increase the satisfactory degree of the leader at the expense of the satisfactory degree of the follower, then he/she increases the minimal acceptable satisfactory level μ_1^*. Conversely, if the leader judges that it is desirable to increase the satisfactory degree of the follower at the expense of the satisfactory degree of the leader, then he/she decreases the minimal acceptable satisfactory level μ_1^*.

The follower should also be treated fairly, although he/she is in a subordinate situation. So, after the leader sets the minimal acceptable reference level μ_1^*, the follower sets the minimal acceptable satisfactory level μ_2^* too.

Now, we give the steps of the interactive fuzzy decision making algorithm to derive a satisfactory efficient solution for the leader and the follower as follows:

Algorithm 1.

Step 1. Determine F_j^L and F_j^U as in (3) and (4).
Step 2. The leader sets the initial minimal acceptable satisfactory level μ_1^*.
Step 3. The follower sets the initial minimal acceptable satisfactory level μ_2^*.

Step 4. Solve problem (6). If there does not exist a solution to (6), the leader or/and the follower reduces his/her or/and their minimal acceptable satisfactory levels, until a solution (x^*, y^*) is obtained for (6).

Step 5. If the leader satisfies the obtained solution (x^*, y^*), then stop, and (x^*, y^*) is a satisfactory efficient solution for the leader and the follower. Otherwise, go to Step 6.

Step 6. The leader and the follower revise the minimal acceptable satisfactory levels μ_1^* or/and μ_2^*, and go to Step 4.

4. Numerical results

In order to measure the satisfaction with a solution from multi-aspect, we not only consider the value of the overall satisfactory degree (i.e., $d(x,y)$) but also the value of distance function $D = \{\sum_0^k [1-\mu(F_i)]^2\}^{\frac{1}{2}}$ (for details, see [32,21]) where $\mu(F_i)$ represents the achieved membership value of the ith decision maker.

We choose two examples to illustrate the feasibility of the proposed method and numerical results are as follows:

Example 1. [25]

$$
\begin{aligned}
&\max_x F_1(x,y) = 5x_1 + 2x_2 + 4x_3\\
&\text{where for a given } x = (x_1, x_2)^T,\ y = (x_3, x_4)^T \text{ solves,}\\
&\max_y F_2(x,y) = 3x_2 + 5x_3 - 2x_4\\
&\text{s.t.}\quad 2x_1 + 2x_2 + 2x_3 + 4x_4 \leqslant 8,\\
&\qquad x_1 + x_2 + x_3 \leqslant 2,\\
&\qquad x_2 + x_3 + x_4 \leqslant 3,\\
&\qquad x_1 \leqslant 4,\ x_2 \leqslant 4,\\
&\qquad x_3 \leqslant 2,\ x_4 \leqslant 2,\\
&\qquad x_1, x_2, x_3, x_4 \geqslant 0.
\end{aligned}
$$

The ideal value and tolerance limit of the fuzzy goal are $F_1^L = 10$, $F_2^L = 10$ and $F_1^U = 0$, $F_2^U = -4$, respectively. Moreover, the proposed method in this paper is denoted by Method 1, and the method of Arora and Gupta [25] by Method 2. In addition, we make a comparison with the results from [25] in Table 1. From the evaluated results of $d(x,y)$ and D, the obtained solution of our method is better than Method 2. Furthermore, the whole of the profits (i.e., the sum of the leader's profits and the follower's profits) (i.e., 18) produced by method 1 is greater than that (i.e., 14) produced by method 2. So, these results shows that the proposed method is feasible.

Example 2. [26]

$$
\begin{aligned}
&\max_x F_1(x,y,z) = -18x_1 + 10x_2 + 11y_1 - 11y_2 + 23z_1 + 40z_2,\\
&\max_{y,z} F_2(x,y,z) = -35x_1 - 9x_2 + 20y_1 - 44y_2 + 10z_1 + 7z_2,\\
&\text{s.t.}\quad 47x_1 - 14x_2 - y_1 + 4y_2 + z_1 - 49z_2 \leqslant 1.5,\\
&\qquad -23x_1 + 2x_2 + 45y_1 - 35y_2 + 12z_1 + 41z_2 \leqslant 13.5,\\
&\qquad -9x_1 - 18x_2 + 12y_1 + 13y_2 + 37z_1 - 11z_2 \leqslant 5.5,\\
&\qquad 6x_1 - 19x_2 - y_1 - 2y_2 - 49z_1 - 11z_2 \leqslant -43.5,\\
&\qquad -31x_1 - 8x_2 + 2y_1 + 17y_2 + 47z_1 - 25z_2 \leqslant 6.3,\\
&\qquad 46x_1 + 3x_2 - 28y_1 + 17y_2 - 36z_1 - 3z_2 \leqslant 22.5,\\
&\qquad -45x_1 + 34x_2 - 44y_1 + 44y_2 + 16z_1 - 2z_2 \leqslant 17,\\
&\qquad 29x_1 - 13x_2 + 38y_1 + 19y_2 - 2z_1 + 7z_2 \leqslant 39,\\
&\qquad 13x_1 + 10x_2 + 27y_1 - 29y_2 - 49z_1 - 38z_2 \leqslant -38,\\
&\qquad x_i \geqslant 0,\quad y_i \geqslant 0,\quad z_i \geqslant 0,\quad i = 1,2.
\end{aligned}
$$

Table 1
Comparison of results of Example 1.

Method	(x,y)	(F_1, F_2)	$(\mu(F_1), \mu(F_2))$	δ	$d(x,y)$	D
1	(0,0,2,0)	(8,10)	(0.8,1)	1.2500	0.9309	0.2000
2	(1,0,1,0)	(9,5)	(0.9,0.6429)	0.7143	0.7093	0.3709

Table 2
Comparison of results of Example 2.

	Proposed method		Wan et al. [26]	
x	0.8961	1.1275	0.8802	1.0951
y	0.0000	0.0749	0.0000	0.0922
z	1.0477	0.5343	1.0260	0.5481
F_1	39.7888		39.6134	
F_2	−30.5918		−30.6250	
$\mu(F_1)$	0.6060		0.6000	
$\mu(F_2)$	0.6008		0.6000	
δ	0.9914		1	
$d(x,y,z)$	0.5598		0.5571	
D	0.5609		0.5657	

The ideal value and tolerance limit of the fuzzy goal are $F_1^L = 51.311$, $F_2^L = -13.533$ and $F_1^U = 22.067$, $F_2^U = -56.263$, respectively. Then, we make a comparison with the results from Wan et al. [26] in Table 2. It is obvious that the obtained function values of the proposed method are better than that of [26]. In addition, we may conclude that the radio δ in (5) for a solution is not as larger as better.

Numerical results show that the proposed method in this paper has the following interesting features.

- From Tables 1 and 2, we can see that the value of D by the proposed method is smaller than that of other methods.
- From Table 2, it doesn't mean that the larger the value δ in (5), the more satisfactory the solution.

These results would suggest that the proposed method is feasible.

5. Conclusion

In this paper, a new interactive fuzzy decision making method based on the concept of membership function is proposed for solving bilevel programming problem. We take the overall satisfactory balance between the leader and the follower into consideration by introducing a new balance function. Then, a satisfactory solution is obtained. Finally, numerical examples illustrate the feasibility of the proposed method.

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